WALL STREET ON A LATTICE: FINANCE MEETS PHYSICS

By applying theories originally developed to explain the physical world, econophysicists try to help investors understand herding behavior, investment bubbles and market crashes.
“I can calculate the motions of the heavenly bodies, but not the madness of people,” Sir Isaac Newton allegedly said after being caught in the speculative bubble of the South Sea Co., which left him flat broke in the 1720s. Being the father of classical mechanics apparently did not give Newton an investment edge.

To find the first useful connection between physics and financial markets, we have to wait for the groundbreaking research of Louis Bachelier in 1900. The French mathematician was the first to realize that the prices of financial assets can be mathematically modeled as random walks. Bachelier’s analysis predates the Brownian motion theory published in 1905 by Albert Einstein. Despite the similarity in the two scientists’ underlying math, Einstein was unaware of Bachelier’s results. Sadly, the Frenchman’s revolutionary contributions had to wait much longer to be fully recognized, thanks largely to the efforts of Paul Samuelson, who in 1970 became the first American to receive the Nobel Memorial Prize in Economic Sciences. By then the relevance of Brownian motion in finance had been independently rediscovered by physicist M.F.M. Osborne in 1959.

The applications of physics-inspired methods to finance do not stop with Brownian motion. For example, the equation governing heat diffusion is essentially the Black–Scholes equation that is at the basis of modern options pricing. Path integrals — invented by American theoretical physicist Richard Feynman for adding quantum probabilities — have found remarkable applications in the pricing of financial derivatives. And, last but not least, in the past two decades a whole new field, econophysics, has emerged from the application to finance of methods originally invented in statistical mechanics, a branch of physics that studies the emergence of the macroscopic features of a complex system using probability theory. More precisely, statistical mechanics links the macroscopic, thermodynamic properties of a system to its microscopic constituents. In the financial applications envisioned by econophysicists, the market is the macroscopic system while the individual financial agents are the microscopic constituents. Understanding how the principal features of financial markets arise from the microscopic interactions is the main task of econophysics.

Following the Herd

Econophysics has relevance for both investors and researchers. It can shed light on how complex market dynamics emerge from the interaction of agents at a micro level and also can provide valuable insights into risk management. In this article we plan to outline some practical contributions of econophysics to financial modeling. They range from studying the herding behavior of market participants — a typical trait of speculative bubbles — to predicting financial crashes. From a statistical physics point of view, all the models we will be introducing have in common the existence of a phase transition that describes the shift between different market regimes. This idea will be elucidated in very simple terms with the introduction of the Ising model — a landmark of statistical mechanics. We will use the Ising model both to introduce phase transitions and to analyze herd behavior.

Herding is a key theme in behavioral finance and corresponds to the tendency of a market participant to imitate the actions of a larger group. Remember the dot-com bubble in the late 1990s, when investors plowed their money into the shares of Internet companies and other tech stocks irrespective of valuations? We will attempt to study this behavior in terms of a simple statistical model.

Picture financial agents as a network, with each point of the network representing a trader. The traders base their trading strategies on their own idiosyncratic signals and on their communications with other traders in the network. An idiosyncratic signal is the result of a trader’s independent analysis and emerges from, for example, back-testing strategies on historical data or the arrival of fundamental news.

Market Without Drift

The second component comes from assessing the opinions of other traders in the network. This piece is particularly relevant when the idiosyncratic signal is weak. In this case traders are more inclined to take the temperature of the market and follow
the flow by polling the opinions of their neighbors in the network. The net effect of this polling is the emergence of a herd or crowd of financial agents that share the same opinion.

Let us try to model this phenomenon by representing the network as a two-dimensional square lattice. Traders are sitting at the vertices of the lattice and have two possible states: +1 (buy) and -1 (sell). If, for the sake of simplicity, we geometrically represent the network as a two-dimensional square lattice, then a typical configuration would appear as in Figure 1. In this figure, orange dots represent traders who have reached the decision to buy; brown dots represent traders wishing to sell. Overall, buyers and sellers tend to balance out one another — this corresponds to a market without drift.

Figure 1: This lattice represents the state of a network of traders when the interaction strength ($K$) is below its critical value ($K_c$). Neither the orange (buy) nor the brown (sell) dots dominate. The market has no drift.

Now consider a trader in the network; we’ll call her Alice. She is aware that to profit from her strategies she has to guess the imbalance between sellers and buyers, as the price change is going to be proportional to the excess demand. If Alice could poll all the traders in the network, she could sum their states together and figure out the overall market sentiment. If the result was bullish, she would choose her position as +1. Analogously, she would position herself as -1 if the sentiment was bearish. In practice, Alice will not possibly know all the traders in the network, so her next best action is to poll her neighbors. We can measure the degree to which Alice aligns her decisions with her neighbors’ sentiment with a positive parameter, $K$. The larger $K$ is, the more likely that Alice will follow the flow.

**The Ising Model**

This kind of interaction is known in statistical physics as the Ising model. It was originally introduced to study magnetic materials. In the physical interpretation each trader would be replaced by a little magnet that could point either south or north. The presence of a market consensus would correspond to a nonvanishing overall magnetization of the system. A net magnetization arises when the majority of magnets point in either a southerly or a northerly direction.

Despite its simplicity, the Ising model has a wonderfully rich behavior. What makes it especially remarkable is the existence of phase transitions. Phase transitions correspond to an abrupt change in the macroscopic properties of a statistical system. Probably the most familiar case is the transition between the liquid and gas phases of water at the boiling point. Other important examples include superconductivity (materials with zero electrical resistance) and superfluidity (fluids with zero viscosity). In the context of our model, the transition is from a disordered (paramagnetic) phase in which all traders point randomly in either the +1 or the -1 direction to an ordered (“ferromagnetic”) phase in which almost all traders coalesce in a unanimous direction, which could be either +1 (buy) or -1 (sell).

We can switch between the two phases by suitably tuning the strength parameter $K$, which, as we recall, measures the communication among neighboring traders. For sufficiently weak interactions — that is, $K$ below a critical value, $K_c$ — we are in the

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paramagnetic phase: The herding behavior is not predominant, traders will trust their guts, and they will trade as dictated by their own signals. In this phase we have roughly the same number of buyers and sellers, and the market does not reach a consensus. Graphically, the network looks like Figure 1. When $K$ is greater than $K_c$, we are in the ferromagnetic phase: Imitation forces are so strong that they prevail over idiosyncratic signals. The network eventually coalesces toward either a strongly bullish or a strongly bearish market. In this case graphically the network looks like Figure 2.

When we move from the paramagnetic to the ferromagnetic phase, we spontaneously break the initial symmetry between buyers and sellers. To understand this aspect, let's go back to the physical interpretation of the lattice, where traders are replaced by little magnets. In the paramagnetic phase all the magnets randomly point either north or south. Therefore the microscopic contributions cancel out and we do not have a net macroscopic magnetic field. If we now flip all the magnets, the macroscopic properties do not change.

The operation of flipping the magnets is a “symmetry” of the system during the paramagnetic phase. Broadly speaking, a certain operation is a symmetry when it changes the system while leaving it in an identical state. A simple example is the rotation of a circle around its center. The rotation is a symmetry because it leaves the circle looking the same.

By contrast, in the ferromagnetic phase the majority of the magnets point in a given direction — say, south. This results in a net macroscopic magnetic field pointing in the same direction. If we flip all the magnets as before, the resulting magnetic field would change direction from south to north; the system is no longer invariant under flipping in the ferromagnetic phase. Physicists say that symmetry has been spontaneously broken. In our financial interpretation the macroscopic properties of the market do not change if in the first (paramagnetic) phase all buyers become sellers and all sellers become buyers: The market overall has no consensus. On the other hand, in the final (ferromagnetic) phase the operation of flipping traders’ sentiment would change the macroscopic property of the market from, say, strongly bullish to strongly bearish.

At the boundary between the two phases, we still have approximately the same number of buy and sell clusters. But in contrast to the paramagnetic phase, in the ferromagnetic phase we have clusters of all sizes, ranging from small groups of traders to massively large crowds whose size is comparable to the overall size of the lattice (see Figure 3). The physical origin of this phenomenon lies in the fact that the correlation length — the typical length scale for the interaction — diverges, making the system scale-invariant. The magnetic susceptibility (which in physical terms measures the reaction of the magnets to the application of a small external magnetic field) also diverges with a power-law behavior. As a matter of fact, the presence of power-law behaviors is one of the most important signatures for the onset of a phase transition.

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Figure 3: During the ferromagnetic phase, clusters of buyers (orange dots) and sellers (brown dots) range in size from small groups to large crowds.

When the imitation strength $K$ is sufficiently low, traders behave without coordination and the impact of new information is minimal. As $K$ grows, imitation becomes more and more contagious, and in proximity of the critical point the susceptibility becomes infinite, signaling that the market is extremely sensitive to any external influence: New information can lead to either a market boom or a crash.

Financial Crashes

“There is no cause to worry. The high tide of prosperity will continue.” With hindsight it is easy to dismiss Treasury secretary Andrew Mellon’s statement from September 1929, on the eve of the Great Crash, as mere overconfidence. Certainly, the most extreme conditions in which risk management can be put to the test are those arising when markets collapse. Developing suitable forecasting tools for such events is of enormous interest — both theoretically and practically.

Market crashes are often protean beasts without a unique explanation. The panic selling that led to the October 1929 crash typically was attributed to margin calls, short-selling and the exit of foreign buyers. Some 60 years later the October 1987 crash was blamed on overvaluation, portfolio hedging and program trading. Here we will focus on the important role that positive feedback plays in crashes, leveraging the ideas previously introduced — in particular, the theory of phase transitions in statistical physics. Market crashes will be considered as critical points in the transition between phases. As we’ve seen, markets can exhibit transitions from a normal phase — in which buy and sell orders largely offset each other, preserving equilibrium — to one in which an abnormal synchronization of actions by participants can lead to strongly correlated behavior. This extreme herding results in a speculative bubble, pushing prices significantly above their fundamental values and ultimately leading to a market crash. From this perspective, the emergence of a crash is a purely endogenous phenomenon, arising when a sufficiently large crowd of market participants starts behaving collectively as a single actor. When this crowd decides to sell, a market crash is inevitable.

Let us stress that not all crashes have an endogenous origin. Some are exogenous in nature, triggered by a catastrophic event (a war, a terrorist attack, a natural disaster) that clearly lies outside the financial markets. Still, sometimes markets do crash without an explicit external triggering event. The main lesson of this final section will be that endogenous crashes seem to leave a peculiar fingerprint in the financial time series. This fingerprint is the clue needed to forecast the time of the crash.

The Power of Power Laws

If we analyze the herding behavior of market participants using the Ising model, we expect the presence of a phase transition for a sufficiently large interaction. The critical point of this transition is assumed to correspond to the time of the crash. The main lesson of this final section will be that the hallmark of a phase transition is the existence of a critical point where certain physical quantities exhibit a power-law behavior.

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This was indeed the case for the magnetic susceptibility in the Ising model. Let us now define the hazard rate as the probability of a market crash per unit of time, provided it has not yet happened. We will posit that the hazard rate also has a powerlike behavior in the proximity of a crash. Note that the market price dynamics are linked to the hazard rate, as the higher the probability of a crash, the higher the compensation demanded by a trader for holding the risk. This, in turn, implies that the logarithm of the price approaches the crash with a powerlike behavior:

\[ \log (\text{price}(t)) \sim A + B (t - t_c)^\alpha \]

where \( t_c \) is the most likely time for the market’s crash.

Phenomenologically, it has been observed that before crashes the market price exhibits oscillations that decorate the powerlike behavior in the equation above. These movements, known as log-periodic oscillations, accelerate as we approach a crash, while becoming smaller and smaller in amplitude (see Figure 4). Their name stems from the fact that the frequency of the oscillations depends on the logarithm of the time to the crash. Log-periodic oscillations have been used in different contexts, such as the study of material fractures and earthquakes. For example, the observation of vibrations with a log-periodic pattern can help predict ruptures in the high-pressure Kevlar tanks used in space rockets.

Figure 4: The log-periodic oscillations (in green) increase in frequency as we approach a critical crash time.

From a practical point of view, the presence of these oscillations is an important signature for an impending crash and allows for a better fit to the data and a more precise estimate of the time of the crash. The academic literature reports the existence of log-periodic oscillations in the proximity of a variety of major market crashes (including 1929, 1987 and the financial crisis of 2008–09). The first concrete application occurred in 1997, when two independent groups of researchers predicted a dramatic crash in October of that year — which, indeed, happened! On October 27 the Dow Jones Industrial Average suffered a massive 7.18 percent loss, and the New York Stock Exchange made the controversial decision to halt trading early for the first time in its history.

Despite the accumulated evidence, there are some reservations about the real predictive power of log-periodic fluctuations. For example, the reliability of a prediction depends on how the historical data are fitted. This, in turn, implies an array of possible crash times. Also, some past predictions have turned out to be false alarms. These important caveats do not necessarily invalidate the underlying methodology presented here, but they certainly suggest that further study is necessary. Although the application of econophysics to the prediction of market crashes is still in its infancy, it is an area of research that has the potential to deliver very practical uses. If Sir Isaac Newton had known about it, he might have been able to save a few bucks.
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